Short Chronology on 19th Century Knots, Physics, and Topology

1794 Gauss publishes A Collection of Knots 1833 Gauss's unpublished paper on the integral of linking 1834 Listing creates the word topologie Listing publishes Vorstudien; unpublished isolation of the Listing knot. 1847 1851 Reimann publishes his *Dissertation* on surface connectivity (homology) 1858 Helmholtz, article On Warble Motion Listing and Mobius discover the Mobius Band 1866 Jordan determines theorems on the topology of surfaces in terms of homotopy. 1867 Thomson, first paper On Vortex Motion Maxwell gives equations for knots in 3 dimensions 1868 Maxwell determines 2 dimensional knot projections and movements Thomson, second paper *On Vortex Motion* (empirical homotopy) 1869 Tait, On Knots I - first lecture (1876-77) - Classical Tait Conjectures 1876 1877 Tait classifies alternating knots up to 7 crossings 1884 Kirkmann classifies alternating knots of 9 crossings 1885 Kirkmann classifies alternating knots of 10 crossings 1871 Betti generalizes Reimann's connectivity to n-dimensions 1874 Schafli and Klein establish double covers and the projective plane 1882 Klein's discovery of his Klein bottle 1884 Tait, On Knots II - isolates Locking (Borromeans) as founding knot existence 1885 Tait. On Knots III - investigation and classification of Locking 1887 Jordan publishes theorem on closed curves. 1890 Little classifies alternating knots of 11 crossings Little classifies nonalternating knots of 8 and 9 crossings 1892 Brunn introduces his Brunnian Links (Borromeans) 1895 Poincaré creates a theory of homology 1898 Little classifies nonalternating knots of 11 crossings \_\_\_\_\_ - - - - -1900 Poincaré claims homology theory can distinguish a 3-d sphere ...... 1904 Poincaré finds a counter example, and conjectures on the nonrecognition of the 3-d sphere by homology (Poincaré's 1st HomologyConjecture) 1905 Poincaré reconjectures that a 3-d sphere is distinguished by a trivial fundamental group (Poincaré's 2nd Homotopy Conjecture) 1930 Whitehead's counterexample: Whitehead link (or lock)

Listing writes his thesis *Vorstudien Topologie* (Listing; 1847). Twenty later C. Maxwell notes his thesis and gives his series of conferences on knots and topology to the London Mathematical Society (Maxwell;1869).

In a convergent study in physics, Hermann von Helmholtz's writes Ueber Integrale der hydrodynamischen Gleichungen, welche den Wirbelbewegungen entspreche (Helmholtz;1858) in order to investigate rotational flows (Warble motion)<sup>12</sup>.

<sup>&</sup>lt;sup>12</sup>-Translated by P. G. Tait and published in English under the title "Tait, P. *On Integrals of the Hydrodynamical Equations, which Express Vortex-motion*".

Approximately ten years later, William Thomson (Lord Kelvin) extended his findings in *Vortex Theory Of Atoms* (Thomson-a;1867) and on *Vortex Motion* (Thomson-b;1867). Both articles investigate vortex patterns that form in flows, while attempting to study them mathematically as models. This much said, Kelvin's efforts to create a general theory of the atom were thwarted by the realization that most three-dimensional vortices are dynamically unstable in nature.

*I - On Knots* (Tait,1876) sets out a framework to classify the various modes of knotting mathematically regardless of their stability in a nature. If Tait and Thomson's project had been successful, then *On Knots*, could have been compared to Mendeleïev's 1869 classification of chemical elements in a periodic table. Yet, Kelvin's efforts to create a general theory of the atom were thwarted by the realization that most three-dimensional vortices are dynamically unstable in nature.

**II&III - On Knots** (Tait, 1884&85), abandons the modelization of Nature and begins to investigate the existence of the knot experimentally within a theory of knots itself; Tait first proposes that the existence of the knot – *Beknottedness* – is determined by mirror-symmetry problems (chirality-amphichierality), then reformulates the problem into a conjecture that both symmetry and the existence of the knot is based on *Locking*. A series of *Locking conjectures* are then put forward.

The place of *On Knots* in this history of science remains uncertain. As an overtly scientific research that it undoubtedly is, *On Knots-I* has primarily interested mathematicians and physicists. Indeed, what are today the classic "*Tait Conjectures*" are a retrospective compilation made by a consensus of mathematicians and physicists that remain within this first division of *On Knots*. It does not confront them with the same difficulties as *On Knots-II&III*, sections in which it is not easy to overlook the experimental dimensions of the mathematics as such. Or if this experimental dimension is dismissed, then *On Knots* becomes highly vulnerable to a critique from modern mathematics and physics as being a naïve and pre-scientific work.

It is not my aim here to suggest that these objections are unfounded, but rather that our first task in reading *On Knots* is to isolate what, if anything, is being systematically overlooked by readers, prior to asking why Tait's project in science and *On Knots* itself possesses the curious privilege of being primitive.

## Plan of Introduction

Our introductions are developed in correspondence with a republication of *On Knots* following its divisions into *I*, *II*, and *III*. We propose a return to *On Knots* by introducing each division with a development of the problems found therein. Hence, this conclusion of *Section I* corresponds to *On Knots I* without limiting its scope to these problems since their development is in anticipation of *On Knots I&II*. Our main focus has been to identify the object of Tait's knot theory without seeking to transpose or assimilate it into the contemporary mathematical and physical theories. We propose that the reader begin to work through *On Knots I*, before turning to our introduction to *On Knots II* as it is

developped in *Section II*. In *Section II*, we will focus on the *mathesis* being developed by Tait in reading and writing the knot. We will focus both on what Tait constructs and how contemporary knot theory has developed such constructions in formal methods ranging from groups to algebraic polynomials. With this hand, we will return in *Section III* to introduce *On Knots III* by developping the object of our investigation in a construction that is closer to the theory of Tait, and as we will show, Lacan.