# Time in Topology and Analysis

ABSTRACT: The question of a 'movement' in topology is habitually expressed in terms of a change over time: that is to say, as succession in space. Thus, the temporal dimension is itself assimilated to a spatial category. Indeed, if one so cares to, one may spatialize time to higher dimensions in the attempt to capture its dynamic aspect in a theory or pure simultaneity. I want to examine this problem by first getting straight on the classic methods, then by way of a critique, we will examine a different manner of constructing a *Topology of Time* as it was first introduced by J. Lacan [Lacan, 1978 ]. 2nd draft proofed by B. Singh

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(Article is still in Pre-Print Form - comments, proofs, or critiques welcome).

## 1- Spatial Categories: Homeomorphism, Homotopy, and Isotopy

In order to come to terms with the problem of time and space in classical topology, it is necessary to respond to the question of what a transformation is; in particular, what is a continuous 1-1 transformation between spaces, i.e., a *homeomorphism*. Once a transformation is determined, and then specified in terms of a *homeomorphism*, we may then proceed to determine the manner in which it can be coded and written into a series of letters or algebra. Such a writing has a name: *homotopy* and *isotopy*. *A Theory of Homotopy/Isotopy* is nothing other than an *Algebraic Theory* that attempts to codify the designs of a *Topology*.

In a first pass, one could make the case that *algebra* is to *digital* as *topology* is to *analogue* and make the comparison to the analytic – or algebraic – geometry of Descartes. Thus, in the table below we show the direction S constitutes an *algebraic topology:*

$$\xrightarrow{\mathbf{S^{-1}}}$$

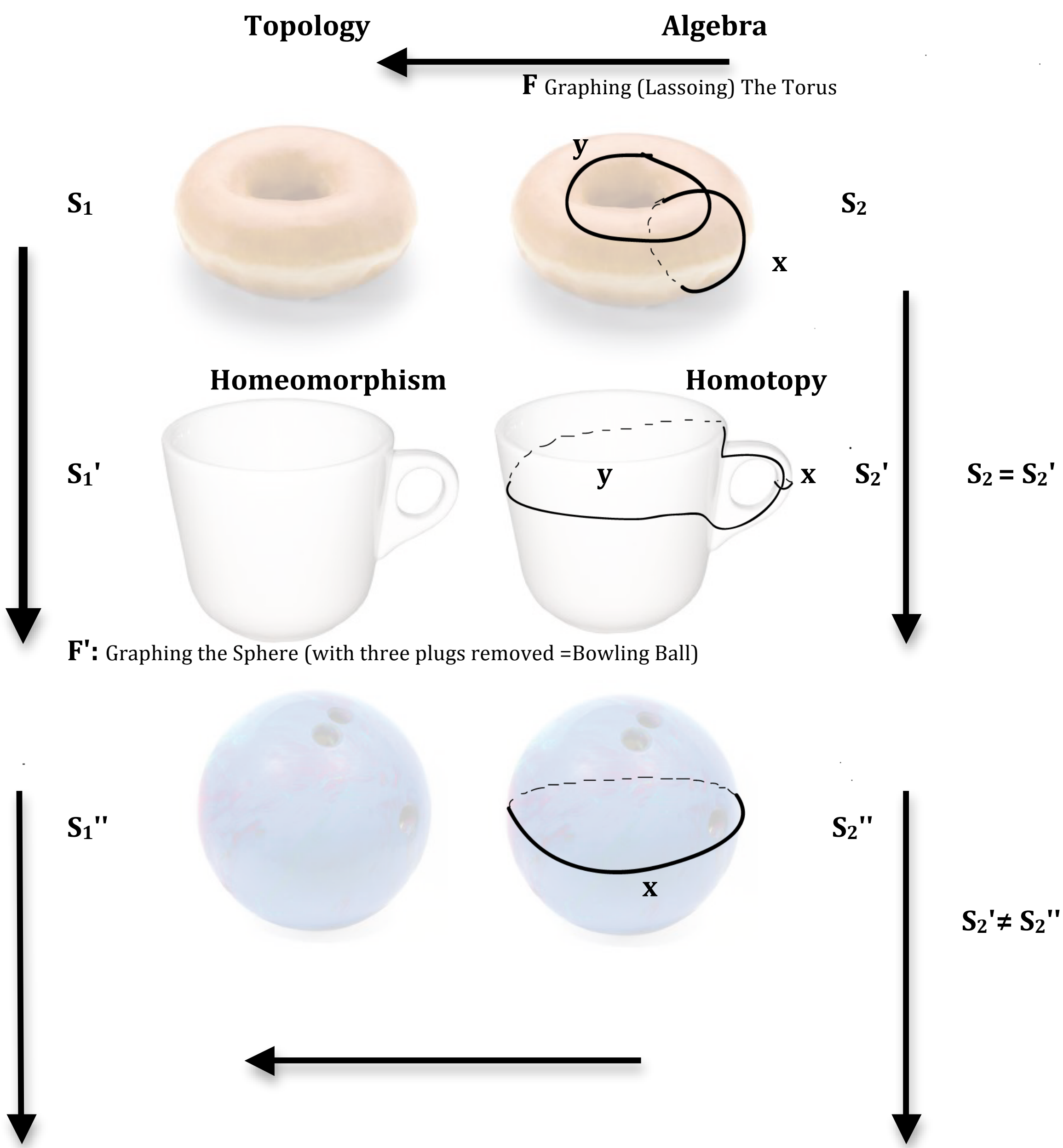
| **Topology** | **Algebra** |
|---|---|
| Traits | Letters |
| Homeomorphism | Homotopy/Isotopy |
| Reading | Writing |

$$\xleftarrow[\mathbf{S}]{}$$

The two directions corresponding to $\mathbf{S^{-1}}$ and **S** correspond to two ways of working with trait and letter that are often confused, which, roughly, can be abbreviated by the difference, respectively, between *topological algebra* and *algebraic topology*. Without clearing up the distinction, we merely state that the direction of **S** may be called *syntactic* in so far as it reduces *writing* to the *codification* of a space within an *algebra,* while the direction $\mathbf{S^{-1}}$ may be called *semantic* in the sense that what the first syntactic direction takes for granted and 'un-definable' in a theory – the intuitions of space as continuity, process, analogue, etc. – the inverse direction attempts to define, i.e., to include the construction of 'un-definables' as itself a problem not of *vision*, but of *reading*, or more precisely of *reading* a *trait* in a theory. If for **S** topology constitutes an *object* – topologies, neighborhoods, limits, borders, etc. – for $\mathbf{S^{-1}}$ Topology introduces a *subject* that has been traditionally defined as the *doctrine* or informal *deductive framework* of the practicing mathematician.

Without clarifying the distinctions further here, let us begin with the more commonplace direction of an **S**-theory or *algebraic topology.*

We may speak of identifying and classifying the various transformations of a space in an algebraic theory of *homotopy-isotopy*. For instance, given the topological object on the left, a torus $\mathbf{S_1}$, it can be put into a 1-1 continuous transformation into another space $\mathbf{S_2}$. However, to rigorously show that this is so, that is to say, to actually read the transformation and not intuit it, we need to determine a graph: cover the torus with a series of 1-dimensional laces – or graphs – then letter the cells such that this writing univocally identifies the object and distinguishes it from other topological surfaces. An *algebraic topologist* can be compared to a blind man who uses a system of lassos (graphs) to determine what kind of objects he is dealing with: a lasso snags a hole in the case of a doughnut (**x,y** below), but with a bowling ball the lasso does not snag. Thus, from the non-point of view of the blind man with a lasso, the two are not equivalent.



The diagram above illustrates a homeomorphism in the vertical passage from two topological spaces demonstrating the popular saying, "*A topologist cannot tell the difference between a doughnut and a coffee cup*". But we must also admit that a pure topologist cannot tell the difference between a doughnut and a bowling ball if s/he does not have any way to prove that one cannot transform one into the other by a *homeomorphism*. Said otherwise, just because a topologist has not yet transformed a doughnut into a bowling ball by a 1-1 continuous transformation (no tearing), does not mean s/he cannot. Indeed, to give a proof that it is impossible implies the use of a writing – or algebra. Thus, we must include the horizontal dimension in our illustration: only an *algebraic topologist* can tell if a torus is not a bowling ball on the condition that s/he determines a graph that codifies the complete space and allows us not only to identify an object, but to tell when two objects are different. What is *invariant* in the movement between the two different presentations of the torus as doughnut and coffee cup is the *homotopy graph*. We will show in a less illustrative way later that it is precisely the sameness of the *graph* of $\mathbf{S_2}$,$\mathbf{S_2'}$ and the difference from the *graph* of $\mathbf{S_2''}$ that rigorously distinguish one from the other in the sense that it *calculates their traits*. For the moment, it suffices to say that a *topological homeomorphism* is between two *spaces*, whereas an *algebraic homotopy* is between two *graphs*, or we will say later, two *functions.*

*Psychoanalysis And Topology*

Although this horizontal passage from *topology* to *algebra, from homeomorphism to homotopy-isotopy* is habitually spoken of in phenomenological and *descriptive* terms as the passage from *quality* to *quantity*, or *analog* to *digital*, we prefer to designate it here in analytic and *inscriptive* terms as the passage from *signifier* to *letter*. We take this position not arbitrarily since most are familiar with the fact that although a topologist may be completely blind (Dr. Molleneaux, Bernard Morin, etc.), it does not seem possible to be without a voice or its inscription into something like a letter.

So, why should these considerations interest the psychoanalyst in the least? Well, if it were not for the first steps made by Lacan, the question would remain unheard of. Today, there are two possible responses: either, that what is called '*Lacanian Topology'* is a confusing excursion of an aging old man that has little or nothing to do with analysis (or anything else for that matter), or that a practice and theory of topology clarifies and makes problems of symbolism effective.

For instance, it is often said, *'Psychoanalysis sees sex everywhere'*. Thus, today anyone who has watched a Woody Allen movie knows that anything from a cave door to an oven maybe taken as a symbol for the vagina. But is this really Freudian? Or is it more of a type of therapeutic soup that has been served up by the cultural commerce of psychoanalysis for over a hundred years? Our position is: not only is this psycho-cultural soup not Freudian, but it confuses reasoning by resemblance – or analogy – with the real process of symbolization (*Bildung*). Indeed, if we proceed on a Freudian basis, then it must be admitted that psychoanalysis 'sees' sexuality nowhere; in the sense that it is not a theory of sexuality that would confuse a tall building with a symbol of a phallus. Rather, psychoanalysis in the manner of Freud attempts to *read* a symbol and recognizes that the present *symbolizer* (the building) has no more resemblance to the absent *symbolized* (the phallus), than the letter 'b' in the word 'building' has to an actual building. In fact, it is only with a reference to a letter and the writing of a trait that the analytic symbol functions to introduce a discontinuity with respect to any representation by resemblance. Again, the same may be said with regard to the topological diagrams above: by reasoning by *analogy,* a coffee cup may always be compared to or used as a symbol to stand for something else. Likewise, a doughnut, tire, or ring may be used as an illustration, model, or analogy for a torus. But it is only by introducing a letter and the writing of a trait – or graph – that it is possible to state this diagrammatic correspondence with precision. Thus, what is necessary in a first rectification is not to remain, whether in reference to myth or science, at the level of spoken *analogies*, but to turn to a more structural and written *homology* or *homotopy*[1].

In opening up this non-representational aspect of the symbol, Freud makes room for the dynamic problem of repression and the sexual drives where the symbol is not viewed as a completed entity or as an analogy between a signifier and signified, but rather as a *work* of symbolization (*Wirlichkeit*). Here, the ramifications for a topology of time enters as a question of what is a *pulsional motion* (*Triebregung*)?

For example, in the process of symbolization by Little Hans, he refuses to move across the street because there is an anguish over a horse, which Freud defines as a *pulsional motion*: "*The pulsional motion is a hostile impulsion directed towards the father*" (Freud, 1926, p. 102)[2]. The existence of this motion is revealed by his wish, recognized through an analysis, to see the horse – a substitute symbolizing the father – fall and be hurt. But Freud also admits a pulsional motion in the opposite direction in the so called 'positive' Oedipus Complex: there exists also, with regard to the father, "*a tender passive motion that represents the desire to be loved by the father as an object in the sense of genital eroticism*". (Freud, 1926, p.105)[3]

In its most simplified form, the question becomes: can an *act* be reduced to a *motion* in space? What is a symbolic act if it is not a univocal, oriented motion in space – a succession of representable moments in time – but an act proceeding in opposite directions? Can we speak of an *'act of structure'* here? And if so, how does such an *act* and *structure* maintain a contradictory relation to representation? Freud calls this non-representational aspect of a symbolization *repression,* then asks what is being repressed? Linear motion? Representation? Or is it the repression of the Symbolizer (Signifier) itself?

As this article attempts to respond to such questions with a topology of time, we only leave here the beginning of a response with a landmark reference to Freud:

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[1] It should be recognized in purely algebraic terms that a *homology* is a 'formal analogy' in the sense that it is a four-part relation like an *analogy*, but that it has an inverse.
[2] My translation.
[3] My translation.

> Until now, we have treated the repression of a pulsional representative in understanding by this last expression, a representation or group of representations invested with a determined quantum of psychic energy (libido, interest). Clinical observation obliges us to decompose what we have so far considered as a whole: it shows us in effect that it is necessary to consider, on the side of representation, something else that represents the pulsion and that this something else is submitted to a pulsional destination that is totally different from that of representation. To designate this other element of the psychic representative, the name of *quantum of affect* is admitted; it corresponds to the pulsion in so far as it is detached from representation, while its expression conforms to its quantity in the process that is experienced under the form of affects.
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> (Freud, 1915, p. 152)[4]

To speak of a *Topology of Time* is to construct a topological account of the problem of repression, while viewing the delicate question of determining a transformation not simply as a *homeomorphism* – a 1-1 continuous transformation – tracked by *homotopies*, *isotopies*, and *homologies*, but as a more discontinuous *heteromorphism* of the pulsion that classical *algebraic topology*– in the sense of **S –** takes for granted. We will define this new movement later when we show how a *Topology of Time* gives rise to a *Topological Algebra* and a *Topology of Subject* noted in the passage noted $\mathbf{S}^{-1}$ at the beginning of this section.

## References

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[4] My Translation.