3§ - Why is Badiou's set philosophy *Pre-Cantorian*?

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ABSTRACT: This section of our investigation broadens the implications of the previous section 2§: *Undefined Matheme: from Stenography to Abbreviation*.

*By 'aggregate'(*Menge*) we understand any collection into a whole (*Zusammenfassung zu einem Ganzen*) M of definite, well differentiated objects m of our intuition or thought. We call these 'elements' of M. In signs we express thus:*

$$M = \{m\}$$

Cantor

We will not be driven out of the paradise Cantor has created for us.

Hilbert

The unconscious is structured as assemblies which in the theory of sets are like letters.

Lacan

If one were pressed for time, and one eventually is, a stenographic use of the *matheme* may well be pardonable, if it were not for the havoc it wreaks once one attempts to construct a theory rigorously. These difficulties were only briefly touched upon in our last chapter with reference to Badiou's *LEE*. My concern in this chapter is to extend our initial archaeological investigation by turning towards how *LEE* interprets set theory; or more precisely, avoids constructing the mathematical basis of such an interpretation *in theory*. Without entering into the claims being made for a philosophical interpretation of set theory – which may indeed change from one publication to another – I propose a reading that is less committed to the programmatic statements of *LEE* and the fashions of the hour. In the next chapters, I will be more focused on exactly what the method of *LEE* reveals. It seems to me that not only is such an investigation more fundamental, but it would serve as a much needed aid to those currently trying to come to terms with the contemporary intersections of philosophy, mathematics, psychology, and psychoanalysis. Indeed, in beginning to read *LEE* from the beginning as an *Event*, as that remarkable book written by Alain Badiou, and not just that such and such a book written by a so and so in the tradition, we will be performing a reading that says more about what his method actually accomplishes, than what the ontology he supports would let him believe. Thus, before going further into the set theoretical constructions here, it will perhaps be useful to examine these beliefs by taking a short detour by the 'intended' interpretations of *LEE*.

By a coincidence that is not altogether avoidable, just as Badiou does not define or calculate a *matheme* (Part I-§2), he does not ever define or deduce a *set*. As a consequence, the identification and existence of a set are never tied down from the beginning with the means disposable to a theory. What follows is a brief argument indicating how an avoidance of a precise writing of the *matheme* leads Badiou to develop a *Pre-*, if not *Non-Cantorian* set philosophy.

Such a conclusion is not that surprising, as it is partly admitted by Badiou himself when he adopts the system of Zermelo-Fraenkel (system ZF) as both exemplary of his ontological approach and a resolution of the aporias of Cantor's naïve set theory. According to Badiou, not only is it necessary for the successors of Cantor to (i) forbid any definition of a set, but (ii) prohibit the formation of "paradoxical multiplicities" [1; 54]. For it is this double limitation that Badiou views as founding the Cantorian tradition:

> This double task [(i) and (ii)] was, between 1908 and 1940, taken up by Zermelo and achieved by Fraenkel, von Neumann and Gödel. Its achievement is the formal axiomatic system where, in a logic of the first order, the pure doctrine of the multiple is presented such that today it can still serve to lay out all the branches of mathematics.
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> I insist on the fact that in what concerns the theory of sets, the axiomatization is not an artifice of exposition, but an intrinsic necessity. [1; 54]

A crucial question remains: by neither defining a *set* or a *matheme* rigorously beforehand, what is actually being talked about, or rather what is to differentiate Badiou's verbal claim for axiomatic set theory as the basis of an ontology as anything more than a dogmatism of the Master or the delirium of Schreber?

I will not respond to this question here, leaving it to the reader to come to his or her own conclusions. Rather my concern is to begin by clarifying what is at stake in dis-assimilating both the theory of the *matheme* and *set* from the historical and philosophical use Badiou tries to make of them.

To read Badiou according to the claims he sets for *LEE*, supposes that we read the text as a philosopher, or at least with a certain innocence, and put both a rigorous definition of a *matheme* and a *set* into suspense. At least at the beginning, for we are advised in *LEE* that a nomination, thus, a possible definition of a set, can only arise, après coup, in the second half of *LEE* not dedicated to *Being*, but *Non-being* and the *Event*. Indeed, it is only in the last chapters of *LEE* that we begin to discover the problem of nomination coming out in a stenography of *mathemes*: the '*matheme*' of the *event,* '*matheme*' of *infinity*, and '*matheme*' of *indiscernibility*.

The question remaining is two-fold: to what extent, if any, does any of this have to do with set theory and a *strict usage* of the *matheme*. If it does, then precisely what is it. If not, then at what point, can such philosophical metaphors be viewed as 'ways of speaking' – or symptoms of being – about what is constantly not being written? Without wishing here to deny how important a symptom or fantasy may be, they are only a preliminary and auxiliary to a more fundamental construction of a theory of writing and the signifier. In order to make room for such a theory, let us try to excavate what the text is committed to, at least in speech, and to determine in what respect this accords with what can and has been written in the name of set theory.

In reading *LEE* it is evident that the reader is being asked to believe that the text – and presumably the philosophy of the author himself – stands in the heritage of Cantorian set theory. The reasons given for this assumption are scant, but it seems to be that Badiou believes because Zermelo's brand of axiomatic set theory is capable of "*laying out all of modern mathematics*" that it shares the philosophical commitments of *LEE* on the one and the multiple. As a consequence, there is the implication of a shared tradition. Once this suggestion is bolstered up with philosophical references, it is supposed to follow quite naturally that the philosophy that Badiou is trying to develop is itself in the Cantorian tradition and that he has somehow managed to go beyond those 'other' set theories: those of the sophists, the nominalists, the intuitionists, and Lacan's "pre-Cantorian" set theory [1-b; *Conditions*, p.298]. Whatever the merits of hearing such echoes in order to get a hold of one's self, the pros and cons have been commented upon at length in literary, philosophical, and artistic circles. In order not to enter into the cave, I will just call out from the opening: I agree with all those who charge that any good grad student in mathematics can philosophize a mathematical result, but this does not mean that just anyone can become a Badiou! This being said, the question remains as to whether any of this can justifiably become serious[1].

It is obvious to even an amateur of set theory that the philosophical set theory being developed by Badiou in *LEE* is based on *Pre-,* if not *Non-Cantorian* principles. The reason is simple: the attribute of being *Pre-Cantorian* follows from the fact that Zermelo's set theory (ZF) is itself *Pre-Cantorian*. In short, the ontology of Badiou and the method of Zermelo are far from the theology and method of Cantor. The divergence between Zermelo's and Cantor's theory has been noticed by more than one. Bourbaki's set theory, for example, maps the tradition of Cantor via Hilbert and begins by not only proposing to define a set, but to no longer limit the paradoxes according to the separation axiom (*Axiom der Aussonderung*) of Zermelo[2]. Thus, contradicting both the tasks (i) and (ii) that Badiou sets for the task of a 'Cantorian' heritage of an ontology:

> "We shall consider that [set theory] principally due to Von Neumann [43; a and b] which comes closer than the Zermelo-Fraenkel system to Cantor's primitive conception; the latter in order to avoid "paradoxical sets, had already proposed (in his correspondence with Dedekind; [25], pp. 445-448) to distinguish two types of sets, "multiplicities" (*Vielheiten*) and "sets" (*Mengen*) in the strict sense, the latter being characterized by the fact they could be thought of as single objects."
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> *Bourbaki* [2]; English Translation; p.330

To go further into the historical details would take us past the limits of this introduction; besides, it is a relatively trivial exercise in mathematical scholarship. Thus, we refer the reader to the exercises at the end of this chapter and any good source book in set theory for the particulars[3].

If one were prone to seeing any merit in such classificatory schemes, the epitaph of 'Cantorian', in the sense of Hilbert [6-a, b], would be more appropriately assigned to the manner Lacan develops set theory with reference to the *tau-square* calculus of Bourbaki (see *Exercises* below). What is odd, or at least inverted, is that Badiou would not read this writing of Lacan, but hear the echo of a voice that would help him, Badiou, write a pre-Cantorian philosophy in spite of his commitments in speech. No doubt, it is always 'the other' who is 'pre-Cantorian': *Intuitionism*, *Structuralism*, *Nominalism*, *Lacanianism*, *Tutti-quanti*. In such misrecognitions, one is reminded of Freud's hallmark of denegation: "*You want to say it is my mother, but I assure you it is not*".

Be that as it may, my argument is neither with the erudition nor the philosophical claims of *LEE*. I am not even opposed to the clinical dimension of 'hearing voices' and 'speaking in tongues' as an intrinsic accompaniment to the writing of mathematics. But the question being proposed is how not to remain there. Before attempting to draw the philosophical implications of set theory, the most important question to be raised is: how does one identify a theory of sets to begin with and does Badiou's *LEE* capture it? If so, how, and if not, then why not and how can these observations help in the construction of a more adequate theory? It is this more direct introduction that I am aiming to write. But first, a few exercises and remarks will close off these initial archaeological excavations.

I-Introductory Exercises and Remarks[4]

1-Remarks: What every schoolchild should know.

a) *Philosophical Set Theory*: assume a set is left undefined and the membership relation '$\in$' is taken as a primitive, then limit its size in the attempt to prohibit sets that are 'too large' sets – actual infinities – such as those that generate the paradoxes of Russell and Cantor. Call this manner of axomatizing and limiting the extension of the term 'set' a philosophical and axiomatic set theory. In such methods, a definition of a set is avoided, while its properties are only vaguely axiomatized via the limitations of size of the type proposed by Russell and Zermelo. The relation of such 'abstract' axiomatizations to the tradition of Cantor's naïve set theory is at best negative – or *Non-Cantorian* – in so far as it is only by a limitation of symptoms that such measures proceed.

b) *Structural Set Theory*: assume the problem of defining a set is not bypassed and the membership relation '$\in$' can not be taken as primitive, but is a *structure* that must be defined in terms of other *mathemes* or abbreviator symbols – *tau* and $\square$, for example. As a consequence, the limitation of size and problems of infinity are secondary, to the actual finite definition, inscription, and deduction of the *structure* of a set. In short, in beginning with (b), one is *forced* to define – or *model* – sets out of pre-set theoretical domains or collections, then deduce their characteristics – or *structure* – from principles that are not mere axiomatic assumptions. In this respect, a set can only be effectively determined by the axioms of a theory to the extent that they can be satisfied in a *model* and/or deduced in terms of the theorems. By such a method, we are deducing what is significant of the term 'set' not simply on the basis of what it is, but what is not, i.e., what can can not be deducible as a non-set. Said otherwise, the problem is not merely one of setting down axioms that stenogram the intuitive properties of a set or limiting the paradoxes, but of determining what is definable and deducible in theorems stated in *terms* signifying what can be called a *set* or *non-set*. As such a theory requires the construction of principles for the deduction of theorems of 'set-hood' and 'non-set-hood' irreducible to a mere statement in axioms.

In contradistinction to (a), let us agree to call the combinatorial mode of proceeding by (b) 'Cantorian' in the sense that one begins by defining a set not as an *axiomatic problem of limitation*, but as a *definitional problem of nomination*: (a) seeks to group a collection of objects called a 'set' with regard to properties that remain implicit in the axioms; (b) is forced to *abbreviate* those objects called 'sets' in formulas that are deduced as theorems (or not) and satisfied in models of axioms. Far from being a mere use of natural language, the scriptural conditions for the formation of a set, are necessary for its formation[5]. Indeed, without these scriptural conditions of letters a set could not be deduced as a term of a theory or distinguished from its underlying domain or non-set, i.e. *collection*. Thus, the constant need for Cantor to begin twice by differentiating set (*Menge*) from manifold (*Mannigfaltigkeit*)[3-a], set (*Menge*) from collection (*Vielheiten*) [4], or indeed, set from a variable domain of number[6]. Following (b), the true successors of Cantor's heritage do not merely limit and leave un-defined the notion of a set in axioms (Zermelo, Badiou), but achieve the aporias of abstract set theory in a higher level theory – in both modern model theory (Kreisel, Tarski, Krivine, Cohen, etc.) and structural theory (Lacan, Bourbaki, Lawvere, Maclane, Guitart, Joyal, etc.).

The difference between (a) and (b), *philosophical-axiomatic set theory* and *structural-model theory*, resides in the fact that the latter begins with possibly contingent collections – categories or classes – and proceeds to determine a combinatorial definition of a set which includes not only the power set formation (all possible combinations of elements of a collection), but reduces the paradoxes of 'too large' actual infinities to similar problems of naming contingent collections in the "too small"[7]. Thus, for (b) it is requisite to develop a system of writing in which the problem of the construction of a mathematical model or object is not to be taken for granted in philosophical speech or a stenographic use of the *matheme*. Indeed, method (a), by not writing its definitions or referential functions precisely, produces three inconsistencies in the construction of a set theory:

a_1) there is no principle by which one can precisely determine an actual infinity because the domain on which one is working has never been determined beforehand.

a_2) the transfinite can not be treated on par with finite[8].

a_3) one can not explain the structure and existence of number – in particular, cardinal, or a purely combinatorial multiplicity[9].

2) Exercise: Call 'Non-Cantorian' any set theory that satisfies description **(a)** and all three conditions noted by **a_1**, **a_2**, **a_3**. Give examples of why both Badiou and Zermeloian set theories can be considered *Non-Cantorian*. Show how Lacan's outline for a set theory, by adopting the method of Bourbaki's *tau-square*, satisfies none of the three 'Non-Cantorian' conditions. Ask if this historical and classificatory schema is actually displacing a more important opposition. Construct the problem with simple examples. Show how the Borromean lock provides a foundation for a cardinal theory of number.

Notes

1-As of late, in the area in which I now live, Los Angeles, Badiou has taken to doing conferences on set theory in critical theory and art departments of the local universities and preparatory schools. One really has to ask how serious any of this really is or even if there is anyone - or more than one – that can actually verify in anything more than speech his transmission of stenograms? Or at least someone should ask why Badiou would not lecture at the California Technological Institute? What is being bypassed when a lecture on a set theory claiming to trace the ontological organization of "our epoch" has zero auditors or critique at the level of an audience oriented towards technique and science? Is it simply the fault of the 'Other'?

2 Zermelo's statement of the *separation axiom* reads: "*whenever the propositional function P(x) is definite for all elements of a set M, M possess a subset M_P containing as elements precisely those elements x of M for which P(x) is true*".[7; p.202]

3 -One landmark among the many: in the opening lines of Cavaille's monumental, *Remarques sur la formation de la théorie abstraite des ensembles*(1938) we discover the following "*In 1925 Von Neumann concluded a new try at an axiomatization [of the theory of sets] by the putting into relief the irreducible character of the non-categoricity of any axiomatic system that would like to preserve the essentials of the theory,* 'we can do nothing else than to observe that there resides here a new objection counter the theory of sets and that for the instant no way of rehabilitating it is in sight" [*Eine Axiomatisierung der Mengenlehre*, Journ. f. Math., t.154, p.240]. *Thus, these various enterprises have had for a result to provoke a juxtaposition of theories more or less different from the primitive cantorian oeuvre: there will be no longer a theory of sets, but those of Zermelo-Fraenkel, those of Von Neuman, those of Brouwer*. [*Philosophie Mathematiques*, p.26-27]. Also see note 9-below.

[4] -Beginners unfamiliar with the set terminology used in these exercises and remarks should refer to any undergraduate text.

[5] In determining a model of a theory of sets by (b), Cantor's method of abstractionism – defining sets on the basis of its properties – was critiqued by Frege, then gradually replaced by Hilbert and Bourbaki with explicitly written principles that abbreviate and define sets in terms of the selection of a trait from a domain. These principles are not properly speaking 'axioms' based on problems of predication of sets, but principles of deduction and selections of a trait **S** – choice and forcing – with reference to the underlying domain – **A** – of set theory. Needless to say, far from being a mere commodity, or 'linguistic-turn', such principles for the deduction and selection of traits – or signifiers – are crucial to formulate in the writing of a theory of sets. So much so that Bourbaki charges that without such a supplementary system of marks, principles, and variable domain, systems such as ZF are not 'abstract' axiomatic theories, but themselves 'naïve' and 'intuitive' forms of stenography: "*From a naïve point of view, many mathematical entities can be considered as collecitons of "sets" of objects. We do not seek to formalize this notion, and in the formalistic interpretation of what follows the word "set" is to be considered as strictly synonymous with "term". In particular, phrases such as "let X be a set" are, in principle, quite superfluous, since ever letter is a term. Such phrases are introduced only to assist in the intuitive interpretation of the text" [Ibid; p.65]*

[6] -Cantor's letters to Dedekind of June and October 1877 show that he was in the possession of the crucial notion of the power or cardinality of an infinite manifold -*Mannigfaltigkeiten* [Cantor & Dedekind 1937, 25]. The word 'power' or 'cardinal' was taken from projective geometry where it is used to denote a one-to-one projective correspondence. *Mannigfaltigkeiten* dates back at least to *Riemann and Husserl*. In his 1878 article [3-a], Cantor refers to transfinite cardinals not in terms of *Menge*, but *Mannigfaltigkeiten*:

If two well-defined manifolds M and N can be coordinated with each other univocally and completely, element by element (which, if possible in one way, can always happen in many others), we shall employ in the sequel the expression, that those manifolds have the same power or, also, that they are equivalent.

Wenn zwei wohldefinirte Mannigfaltigkeiten M und N sich eindeutigund vollständig, Element für Element, einander zuordnen lassen (was, wenn es auf eine Art möglich ist, immer auch noch auf viele andere Weisen geschehen kann), so möge für das Folgende die Ausdrucksweise gestattet sein, dass diese Mannigfaltigkeiten gleiche Mächtigkeit haben, oder auch, dass sic äquivalent sind."

In the effort to prove a definition of cardinal number within his system of sets (Menge), Zermelo (1904) trivializes it into an ordinal notion necessitating the axiom of choice.

[7] -see Lacan's early development of a logic of collections in *'Le nombre treize et la logique de la suspicion'* in *Autres Écrits,* Editions du Seuil,1946.

[8] - The discovery of the transfinite is only of consequence if Cantor is able to construct a method that can operate on an actual infinity in same manner as the finite. Thus, this method and problem was made precise by Hilbert's *epsilon calculus,* then generalized in Bourbaki's *tau-square* constructions. Such a method is the basis of a set theory that is mathematical in the sense of the *matheme*.

[9] -Following this thesis M. Hallet [5 ;p.4] has stressed the Pre-Cantorian aspects of Zermelo's set theory: "*It follows from Cantor's position that a necessary condition for reductionism (or the reductionist tendency) is that it* must *be able to explain the structure and existence of the numbers. I stress this here because this condition was later abandoned by Zermelo and Fraenkel, who tried to reconstruct set theory without taking the demands of a reductionist theory of number into account. In a sense, this was an attempt to return to the pre-1878 theory, where 'number theory', not just ontologically but in spirit too, is nothing but the comparison of sets. Von Neumann subsequently restored Cantor's condition and this necessitated a considerable extension of Zermelo's system. In essence, von Neumann recreated modern set theory – along more strictly Cantorian lines*".

<u>*References*</u>

[1] Badiou, A.:

a -1988, '*L'être et événement*', Éditions Du Seuil, Paris.

b- 1992, *Conditions*, Éditions Du Seuil, *Paris*.

[2] Bourbaki, N., 2000, *Elements Of Mathematics: Theory Of Sets*, Eng. Trans., p.15.

[3] Cantor, G.:

a- 1878 *Ein Beitrag zur Mannigfaltigkeitslehre*, *Jr M* 84, 242-58. References to [Cantor 1932], 119-133.

b- 1879/84 *Ober unendliche, lineare Punktmannichfaltigkeiten*, *MA* 15, 1-7, 17, 355-58, 20, 113-21, 21, 51-58, 545-86, 23, 453-88. References to [Cantor 1932], 139-244.

c- 1883 *Grundlagen einer allgemeinen Mannigfaltigkeitslehre,* Leipzig, 1883 (separate edition of [Cantor 1879/84], part 5). References to [Cantor 1932], 165-208. English trans, in [Ewald 1996], vol.2.

d -1892 Ober eine elementare Frage der Mannigfaltigkeitslehre, *DMV* 1, 75-78. References to [Cantor 1932], 278-280. English trans, in [Ewald 1996], vol.2.

e- 1895/97 *Beitr/age zur Begrtindung der transfiniten Mengenlehre*, *MA* 46, 481-512, 49, 207-46. References to [Cantor 1932], 282-351.

f-1915 *Contributions to the founding of the Theory of Transfinite Numbers,* Chicago, Open Court. Reprint New York, Dover, 1955.

g- 1932 *Gesammelte Abhandlungen mathematischen und philosophischen lnhalts,* ed. E. Zermelo, Berlin, Springer. Reprint Hildesheim, G. Olms, 1966.

[4] CANTOR, G. & R. DEDEKIND

1937 *Cantor-Dedekind Briefwechsel,* ed. E. Noether and J. Cavailles, Paris, Hermann. French translation including the 1899 letters in [Cavailles 1962], 187-249. English trans, in [Ewald 1996], vol.2.

[5] Hallet, M., 1984, *Cantorian set theory and limitation of size*,

[6] Hilbert, D.:

a-1923, *'Die Logischen Grundlagen der Mathematik', Mathematische Annalen,* 88, 151-165.

b- and Bernays, P. ; 1970, *Grundlagen der Mathematik*, 2nd ed., Springer, Berlin.

[7] Lacan, J., 1975, *Éncore*, Livre XX, (1972-1973); edition established by J.A. Miller, Editions Du Seuil, Paris.

[8] Milner, J.C., 1995, *Oeuvre Claire,* Editions du Seuil.

[9] Zermelo, E., 1908, *Investigations In The Foundations Of Set Theory*, in J. Heijenoort's From Frege to Gödel, 1967; (p.199-215).